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Improvement of Error Covariance Matrices in Data Assimilation

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Introduction

The principle of data assimilation methods consists in finding a compromise between background predictions and instrumental observations where the associated weights are provided by prior error covariance matrices. The background error covariance matrix **B** and the observation error covariance matrix **R** are key elements in data assimilation algorithms. Advanced knowledge of these matrices could be helpful to improve the output error covariance recognition as well as the accuracy of state estimation.

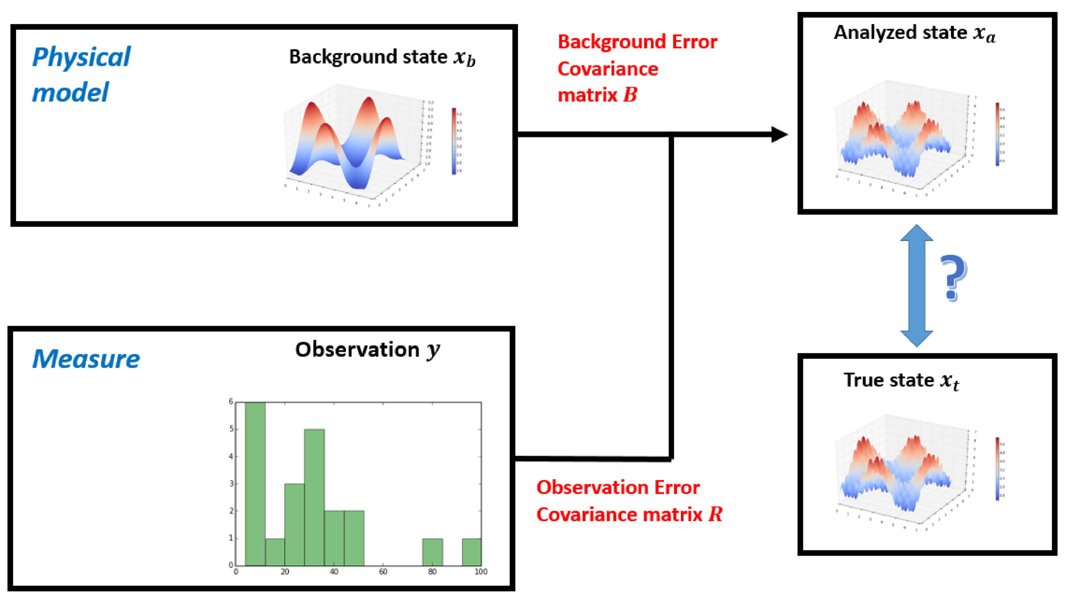

Fig. 1: Data assimilation principle

Continuous attention and effort have been dedicated to this topic, especially for the computation of matrix **B**. The Desroziers iterative method [2] is very well known and widely applied. This method consists in adjusting the ratio between matrices **B** and **R**, supposing the error correlations are well-known. Other existing methods, such as the NMC method [3] or ensemble methods, are more appropriate in a successive data assimilation procedure but not for our approach where we are especially interested in short term predictions and static reconstructions. In this research, we have a dual objective:

- Better identification of *a priori* and *a posteriori* error correlation based on a good knowledge of observation error covariance
- Reduction of prediction/reconstruction error in short term forecasts or static reconstruction

Novel iterative Methods relying on invariant observations

In industrial applications, the model error of reconstruction problems is often integrated as a part of the background error, leading to a less precise knowledge about the background covariance matrix **B** relative to the observation covariance matrix **R**. In addition, [4] points out that an overestimation of matrix **B** could bring an important risk on *a posteriori* error covariance estimation. In order to balance the mis-specification of background state (both $\mathbf{x}_b$ and its covariance **B**), the idea of repeating assimilation loops using well-known observation matrix **R** comes naturally. However, the independence between the background errors and the ones of observations stands for one of the most important hypotheses in classical data assimilation. Therefore, between updated state and observations, the redundancy created by the iterative process itself must be properly estimated and taken into account.

Algorithm: State & Covariance updating
Input: observation data $\mathbf{y}$
Initially guessed matrix: $\mathbf{B}$
Initial state: $\mathbf{x}_b$
for number of updating steps **do**
  Optimization procedure with current $\mathbf{x}_b$ and B
  Estimating the covariance (output state, invariant obervations)
  Updating of covariance matrices and background state
end
Output: structurally improved error covariance matrix and modified state

The optimization step is carried out using either CUTE or PUB algorithms, developed in this work [1].

CUTE (Covariance Updating iTerativE) method

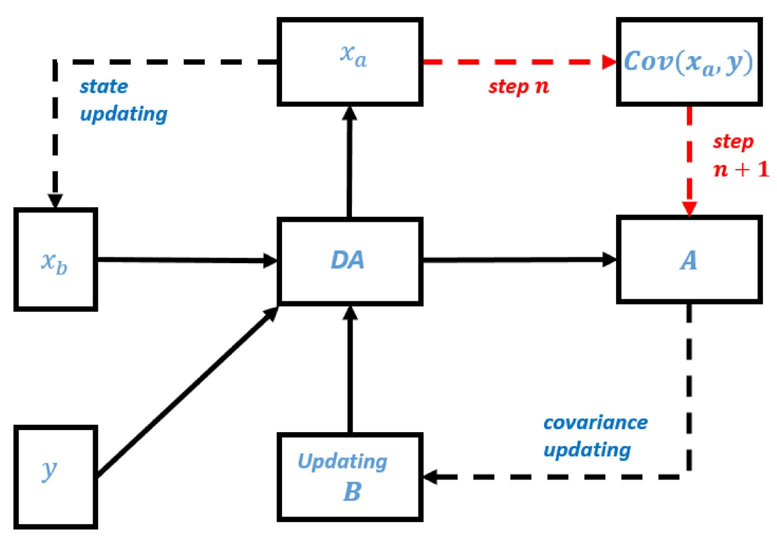

Fig. 2: CUTE algorithm

The covariance between updated states and invariant observations are estimated using BLUE-type formulations and then injected into the estimation of output error covariance in the next loop, as shown by dashed red lines in Fig. 2.

PUB (Partially Updating BLUE) method

The principle of this method is to merge the background state and the observations in a broader space of larger dimension, with a partial updating only on the background part of the space. As a consequence, the updated background-observation redundancy is not only taken into account in the covariance updating but also in the optimization calculation. The covariance updating is based on **BLUE** formula in the broader space.

Twin experiments

We consider a standard 2D shallow-water fluid mechanics system for evaluating the performance of data assimilation algorithms. A cylinder of water, localized in the center of the domain, is released at $t = 0$. The wave-propagation is numerically simulated.

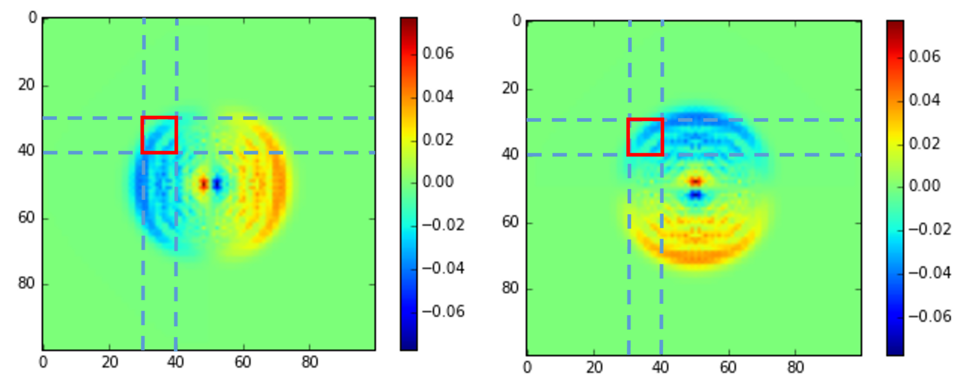

Fig. 3: Shallow water model

The goal of twin experiments is to reconstruct the velocity fields, in a spatial subdomain, based on artificially noised background states and observations.

Reconstruction at a fixed time

We show the prior and the posterior state error correlation functions (correlation value ϕ against distance r) as well as the evolution of assimilation error against the number of iterations in CUTE or PUB methods.

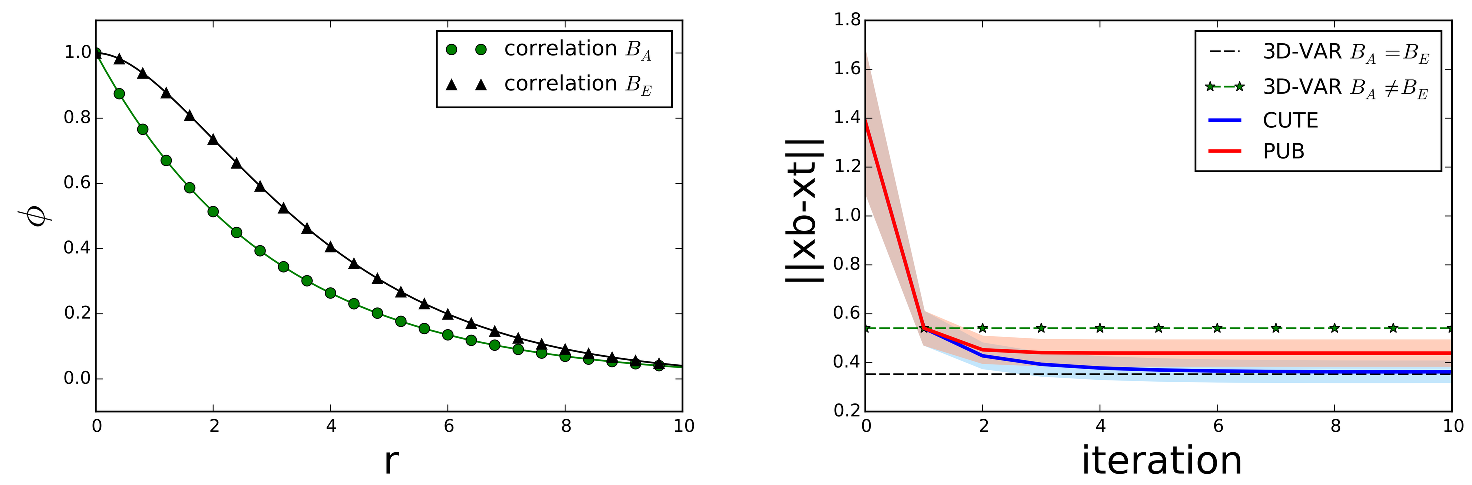

Fig. 4: [left] Original assumed (green) and exact (black) background error correlation; [right] Evolution of reconstruction errors of proposed methods

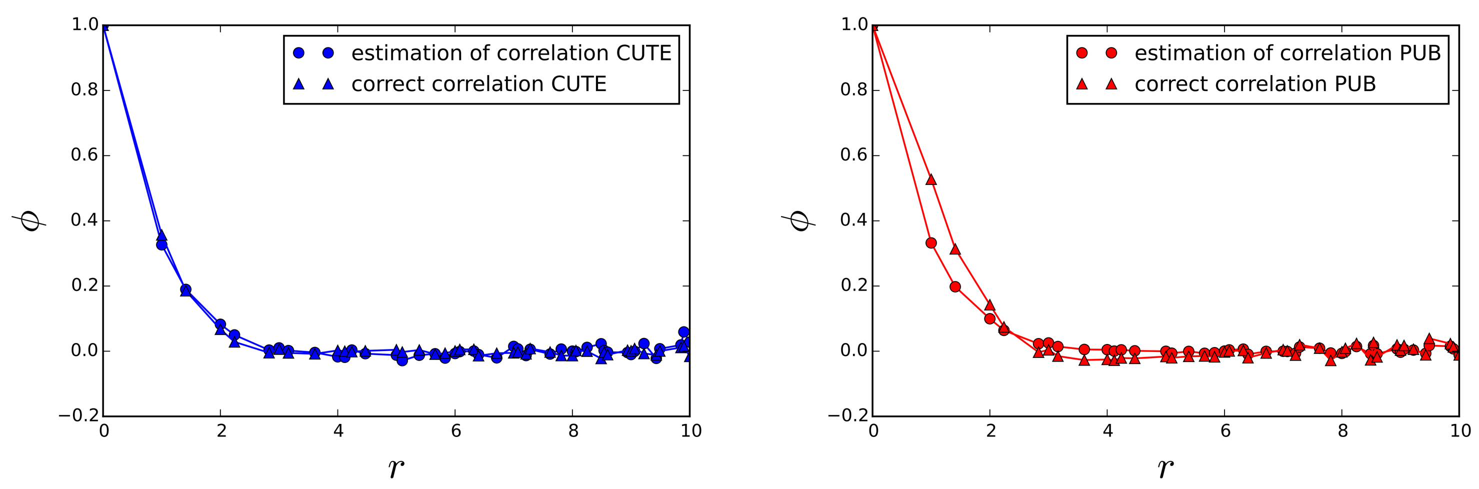

Fig. 5: Estimated compared to exact *a posteriori* error correlation

Conclusions and Perspectives

- Under our assumptions, both CUTE and PUB show strong competitive performance in terms of improving error correlation recognition and assimilation accuracy
- For successive reconstruction in a data assimilation chain (not shown in this poster), improved results are obtained by applying CUTE or PUB method only once at the beginning of the process
- These methods are being tested for a rainflow hydrological problem

References

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