MascotNum2019 conference - Estimation of Borgonovo's moment independent importance measures

Derennes Pierre
Université Paul Sabatier (Toulouse).

Supervisor(s): Prof. J. Morio (ONERA Toulouse) and Prof. F. Simatos (ISAE Toulouse).

Ph.D. expected duration: Sep. 2016 - Jul. 2019

Adress: ONERA - The French Aerospace Lab; 2 Avenue Edouard Belin, 31000 Toulouse.

Email: pierre.derennes@onera.fr

Abstract:

In diverse disciplines, complex systems modeling is often achieved by considering a black-box model for which the observation is expressed as a deterministic function of external parameters representing some physical variables. These basic variables are usually assumed random in order to take phenomenological uncertainties into account. Then, *global sensitivity analysis* (GSA) techniques play a crucial role in the handling of these uncertainties and in the comprehension of the system behavior.

Variance-based sensitivity indices are one of the most widely used GSA measures. They are based on Sobol's indices which express the share of variance of the output that is due to a given input or input combination. However, by definition they only study the impact on the second-order moment of the output which is a restricted representation of the whole output distribution. Moment independent importance measures have been proposed by E. Borgonovo [1] in order to alleviate this drawback. Throughout we consider a general input-output model $Y = \mathcal{M}(X_1, ...X_d)$ where the scalar output Y depends on a d-dimensional real valued random variable $\mathbf{X} = (X_1, ..., X_d)$ through a deterministic function $\mathcal{M}$. We assume that for every $I \subset \{1, \dots, d\}$ a strict subset, the pair $(\mathbf{X}_I, Y)$ is absolutely continuous. The idea of Borgonovo's GSA approach is to measure how fixing $\mathbf{X}_I$ at a value $\mathbf{x}_I$ modifies the entire distribution of the output Y. This modification is quantified by the *shift* $s(\mathbf{x}_I)$ defined from the L_1-norm between the output probability density function (PDF) f_Y and the conditional output PDF $f_Y^{\mathbf{X}_I=\mathbf{x}_I}$:

$$s(\mathbf{x}_I) = \frac{1}{2}\left\| f_Y - f_Y^{\mathbf{X}_I=\mathbf{x}_I} \right\|_{L^1(\mathbb{R})} = \frac{1}{2}\int \left| f_Y(y) - f_Y^{\mathbf{X}_I=\mathbf{x}_I}(y) \right| dy \ . \tag{1}$$

So as to consider the whole range of values the random variable $\mathbf{X}_I$ can take into account, the sensibility of the output Y with respect to the input $\mathbf{X}_I$ is defined by averaging the shift over $\mathbf{X}_I$:

$$\delta_I := \mathbb{E}\left[s(\mathbf{X}_I)\right] \ . \tag{2}$$

Estimating Borgonovo's indices is a challenging task because of the unknown unconditional and conditional PDFs f_Y and $f_Y^{\mathbf{X}_I=\mathbf{x}_I}$ that intervene in a convoluted way (i.e., through an L_1-norm) in their definitions (1) and (2). The estimation of first order indices δ_i has been the subject of extensive investigation in several works, see for instance [4][6][7].

In [6], it is shown that Borgonovo's indices can be reinterpreted as a dependence measure:

$$\delta_I = \frac{1}{2}\left\| f_{\mathbf{X}_I} f_Y - f_{\mathbf{X}_I,Y} \right\|_{L_1(\mathbb{R}^2)} \ . \tag{3}$$

The first contribution of this work [3] consists in introducing a new estimation scheme of δ_i measures from the definition (3). The proposed method combines importance sampling and the

Gaussian kernel estimation of the output PDF f_Y and joint PDF $f_{X_i,Y}$ and enables the estimation of all the first order indices δ_i from one data set. Furthermore, some theoretical properties of the proposed estimator, and in particular its consistency, are derived. However, some limitations exist due to the use of Gaussian kernel estimation which may be inefficient in the case of bounded support or heavy tailed distributions.

The second contribution is to define a new estimation approach avoiding these drawbacks. The starting point is the observation made by [5] that δ_i indices can be expressed in terms of *copula density*:

$$\delta_i = \frac{1}{2} \int_{[0,1]^2} |c_i(u,v) - 1| \, dudv \ . \tag{4}$$

where c_i denotes the copula density of the pair (X_i, Y), i.e, the PDF of $(F_{X_i}(X_i), F_Y(Y))$. In [2], it is proposed to estimate the double integral in Eq.(4) from a simple Monte Carlo estimation coupled to the maximum entropy estimation of the bivariate density copula c_i with *fractional moments* constraints. Some promising results are obtained on different test cases [2].

The application of this work to reliability analysis framework is an interesting perspective. Practitioners often seek to estimate a so-called failure probability associated to an unsafe and undesired state of the considered system. This uncertainty propagation phase may be completed by considering the δ-sensitivity measures of the system conditioned on this failure state. Another perspective is the estimation of higher order δ-indices which relies on being able to estimate high-dimensional PDFs. The difficulty of this problem lies in the *curse of dimensionality.* Indeed the computation burden required for sufficiently accurate estimates obtained with classical methods like kernel estimation and maximum entropy estimation excessively grows when the dimension increases.

References

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Short biography – Pierre Derennes is a second year PhD student of applied mathematics funded by Université Paul Sabatier in Toulouse. His home lab is ONERA (Toulouse). He obtained his master's degree at Université Claude-Bernard in Lyon.